# GCE AS/A level

0981/01

# MATHEMATICS M2
## Mechanics

A.M. TUESDAY, 10 June 2014

1 hour 30 minutes

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Take $g$ as $9{\cdot}8\,\text{ms}^{-2}$.

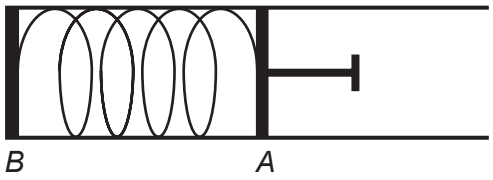
Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** The diagram shows a piston, of mass 0·8 kg, enclosed in a horizontal tube and attached to a light spring of natural length 0·2 m and modulus of elasticity 625 N. The other end of the spring is fixed to the end of the tube at point *B*.




Initially, the piston is held at rest at a point *A* with the spring compressed a distance of 0·1 m, so that *AB* is the compressed length of the spring.

*(a)* Calculate the elastic energy stored in the spring. [2]

The piston is then released. During the subsequent motion, it is subjected to a resistance to motion of constant magnitude 46 N.

*(b)* Determine the velocity of the piston when the spring reaches its natural length. [5]

**2.** A particle of mass 5 kg moves under the action of a horizontal force given by $F = 30t^{-2} - 30$ N at time $t$ s, where $t > 0$. It also experiences a constant resistance to motion of magnitude 120 N.

*(a)* Show that the motion of the particle satisfies the differential equation

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t^{-2} - 30,$$

where $v$ ms$^{-1}$ is the velocity of the particle at time $t$ s. [2]

*(b)* Calculate the value of $t$ when the acceleration of the particle is 24 ms$^{-2}$. [2]

*(c)* Given that the velocity of the particle is 18 ms$^{-1}$ when $t = \frac{1}{3}$, find an expression for $v$ in terms of $t$. Hence find the values of $t$ when $v$ = 10. [6]

**3.** A vehicle of mass 4000 kg is travelling up a slope inclined at an angle $\alpha$ to the horizontal, where $\sin\alpha = \frac{2}{49}$. The engine of the vehicle is working at a constant rate of 90 kW.

*(a)* Calculate the resistance to the motion of the vehicle at the instant when its speed is 4·8 ms$^{-1}$ and its acceleration is 1·2 ms$^{-2}$. [6]

*(b)* Determine the maximum velocity of the vehicle when the resistance to motion has magnitude 12 800 N. [4]

**4.** At time $t$ = 0, an aeroplane $A$ has position vector $(3\mathbf{i} + 5\mathbf{j} + 20\mathbf{k})$ m and is flying with constant velocity $(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ ms$^{-1}$.
At time $t$ = 0, another aeroplane $B$ has position vector $(-2\mathbf{i} + x\mathbf{j} + 15\mathbf{k})$ m, and is flying with constant velocity $(3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k})$ ms$^{-1}$.

*(a)* Find expressions for the position vector of $A$ and the position vector of $B$ at time $t$ s. [3]

*(b)* Determine an expression for $AB^2$, where $AB$ is the distance between $A$ and $B$ at time $t$ s. [4]

*(c)* Given that the shortest distance between $A$ and $B$ occurs at $t$ = 5, calculate the value of $x$. [3]

**5.** A player kicks a ball from a point $A$ on horizontal ground so that 2·5 seconds later the ball just clears a bar at a point $B$. The point $B$ is 3 m above the ground. The horizontal distance of $B$ from $A$ is 42 m.

*(a)* Calculate the horizontal and vertical components of the initial velocity of the ball. [4]

*(b)* Find the magnitude of the velocity of the ball and the angle that the direction of the velocity makes with the horizontal as it passes the point $B$. [6]

*(c)* Determine the horizontal distance from $B$ to the point where the ball first hits the ground again. [3]

**6.** A particle of mass 3 kg moves on a horizontal plane. At time $t$ = 0, the particle has position vector $-2\mathbf{i} + 3\mathbf{j}$ m, where $\mathbf{i}$ and $\mathbf{j}$ are unit vectors along the $x$-axis and $y$-axis respectively. At time $t$ s, the particle moves with velocity $\mathbf{v}$ ms$^{-1}$ given by

$$\mathbf{v} = 4\sin 2t\,\mathbf{i} + 15\cos 5t\,\mathbf{j}.$$

*(a)* Find the magnitude of the force acting on the particle at time $t = \frac{3\pi}{2}$ s. [5]

*(b)* Determine the position vector of the particle at time $t$ s. [4]

*(c)* Calculate the time and the distance of the particle from the origin when it crosses the $y$-axis for the first time. [4]

**7.** One end of a light rod of length $l$ metres is freely jointed to a fixed point $O$ and the other end is attached to a particle of mass $m$ kg. The particle is projected so that it describes a vertical circle. The speed of the particle at the highest point, $u$ ms$^{-1}$, is a quarter of its speed at the lowest point of the circle.

*(a)* Show that $u^2 = \frac{4}{15}gl$. [3]

*(b)* When the rod is inclined at an angle $\theta$ to the **downward** vertical,
(i) find an expression for the tension in the rod in terms of $m$, $g$ and $\theta$.
(ii) determine the value of $\theta$ when the tension in the rod becomes zero. [9]

**END OF PAPER**